KSU CET

S1 & S2 Notes

2019 Scheme

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MODULE 1

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OSCILLATION AND WAVES

HARMONIC OSCILLATION - DEFERENTIAL EQUATION AND IT'S SOLUTION

$F = Ma$

$$= M\frac{d^2x}{dt^2} \quad \text{—— (1)}$$

$F = -kx$ —— (2)

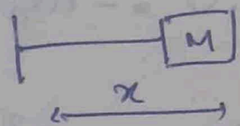
$$M\frac{d^2x}{dt^2} = -kx$$

$\div$ by M

$$\frac{d^2x}{dt^2} = -\frac{kx}{m}$$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0$$



$V = \frac{\text{displacement}}{\text{time}}$

$= \frac{dx}{dt}$

$a = \frac{\text{velocity}}{\text{time}}$

$\frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right)$

$= \frac{d^2x}{dt^2}$

$\omega_0 = \sqrt{\frac{k}{M}}$

$\omega_0^2 = \frac{k}{M}$

This is the differential equation for harmonic motion.

$$\frac{d^2x}{dt^2} + \omega_0^2 x = 0 \text{ ——— (3)} \qquad \frac{dx}{dt} = v$$

$$\frac{d^2x}{dt^2} = -\omega_0^2 x$$

$$\frac{dv}{dt} = -\omega_0^2 x$$

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$$\frac{dv}{dx} \cdot \frac{dx}{dt} = -\omega_0^2 x$$

$$\frac{dv}{dx} \cdot v = -\omega_0^2 x$$

$$v \cdot dv = -\omega_0^2 x \cdot dx$$

Integrating both sides.

$$\int v \cdot dv = \int -\omega_0^2 x \cdot dx$$

$$\frac{v^2}{2} = -\omega_0^2 \frac{x^2}{2} + C \text{ ——— (4)}$$

$x = a$ (amplitude).

at extreme position $v = 0$.

ie, $x = a$, $v = 0$.

$$\frac{0^2}{2} = -\omega_0^2 \frac{a^2}{2} + C$$

$$\omega_0^2 \frac{a^2}{2} = C$$

$$(4) \Rightarrow \frac{v^2}{2} = -\omega_0^2 \frac{x^2}{2} + \frac{\omega_0^2 a^2}{2}.$$

$$\frac{v^2}{\cancel{2}} = \frac{\omega_0^2}{\cancel{2}}(a^2 - x^2)$$

$$v^2 = \omega_0^2 a^2 - \omega_0^2 x^2$$

$$= \omega_0^2 (a^2 - x^2)$$

$$v = \sqrt{\omega_0^2 (a^2 - x^2)}$$

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$$V = \pm \omega_0 \sqrt{a^2 - x^2} \quad \text{—(5)}$$

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$$(5) \Rightarrow \frac{dx}{dt} = \omega_0 \sqrt{a^2 - x^2}$$

$$\frac{dx}{\sqrt{a^2 - x^2}} = \omega_0 . dt$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \int \omega_0 . dt$$

$$\sin^{-1} \frac{x}{a} = \omega_0 t + \phi$$

$$\frac{x}{a} = \sin(\omega_0 t + \phi)$$

$$x = a \sin(\omega_0 t + \phi)$$

solution of differential equation of simple harmonic motion.

{PHYSICAL SIGNIFICANCE OF ω} $\Rightarrow$

when $t \rightarrow \left(t + \frac{2\pi}{\omega_0}\right)$

$$x = a \sin\left(\omega_0 \left(t + \frac{2\pi}{\omega_0}\right) + \phi\right)$$

$$= a \sin(\omega_0 t + 2\pi + \phi)$$

$$V = \frac{\text{displacement}}{\text{time}}$$

$$t = \frac{dis}{V}$$

$$= \frac{2\pi}{\omega_0} \text{ (for a circle)}$$

$$\nu = \frac{1}{T} = \frac{1}{\frac{2\pi}{\omega_0}} = \frac{\omega_0}{2\pi}$$

$$\Longrightarrow \omega_0 = 2\pi\nu$$

Maximum displacement, $x_{max} = A$ because $\sin(\omega_0 t + \phi)$

DAMPED HARMONIC OSCILLATOR

restoring force → dissipative force

The decrease in amplitude of ~~of~~ an oscillator by dissipative force is called damping.

$$M\frac{d^2x}{dt^2} = -kx - r\frac{dx}{dt} \quad \text{—— (1)}$$

r = damping coefficient.

$$M\frac{d^2x}{dt^2} + kx + r\frac{dx}{dt} = 0$$

$\frac{r}{2M} = c$, damping constant

÷ by M

$\frac{r}{M} = 2c$

$$\frac{d^2x}{dt^2} + \frac{k}{m}x + \frac{r}{m}\frac{dx}{dt} = 0.$$

$$\frac{d^2x}{dt^2} + \omega_0^2 x + 2c\frac{dx}{dt} = 0.$$

$$\frac{d^2x}{dt} + 2c\frac{dx}{dt} + \omega_0^2 x = 0. \quad \text{—— (1)}$$

Differential equation of damped harmonic motion.

solution ⟹

Let $x = Ae^{\alpha t}$

$$\frac{dx}{dt} = \frac{d}{dt}Ae^{\alpha t}$$

$$= A\frac{d}{dt}e^{\alpha t} = A\frac{d}{dt}\alpha e^{\alpha t}$$

$$= \underline{A\alpha e^{\alpha t}}$$

$$\frac{d^2x}{dt^2} = \frac{d}{dt} A\alpha e^{\alpha t}$$

$$= A\alpha e^{\alpha t} \frac{d}{dt} \alpha t$$

$$= A\alpha^2 e^{\alpha t}$$

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$$(2) \Rightarrow A\alpha^2 e^{\alpha t} + 2C A\alpha e^{\alpha t} + \omega_0^2 A e^{\alpha t} = 0$$

$$A e^{\alpha t}(\alpha^2 + 2C\alpha + \omega_0^2) = 0. \quad \text{---} (3)$$

ASSUME $\alpha^2 + 2C\alpha + \omega_0^2 = 0.$

$$a = 1,\ b = 2C,\ C = \omega_0^2$$

$$\alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2C \pm \sqrt{(2C)^2 - 4 \times 1 \times \omega_0^2}}{2 \times 1}$$

$$= \frac{-2C \pm \sqrt{4C^2 - 4\omega_0^2}}{2}$$

$$= \frac{-2C \pm \sqrt{4(C^2 - \omega_0^2)}}{2}$$

$$= \frac{-2C \pm 2\sqrt{C^2 - \omega_0^2}}{2}$$

$$= \cancel{2}\left\{\frac{-C \pm \sqrt{C^2 - \omega_0^2}}{\cancel{2}}\right\}$$

$$= -C \pm \sqrt{C^2 - \omega_0^2}$$

$$\alpha_1 = -C + \sqrt{C^2 - \omega_0^2}$$

$$\alpha_2 = -C - \sqrt{C^2 - \omega_0^2}$$

$\therefore \alpha = \alpha_1 + \alpha_2$ (it happens in second order differential equation)

Now,

x - displacement

$$x = Ae^{\alpha t} = A_1 e^{\alpha_1 t} + A_2 e^{\alpha_2 t} \quad \text{——— (4)}$$

$$= A_1 e^{(-c+\sqrt{c^2-\omega_0^2})t} + A_2 e^{(-c-\sqrt{c^2-\omega_0^2})t} \quad \text{——— (5)}$$

$$x = A_1 e^{-ct} e^{\sqrt{c^2-\omega_0^2}t} + A_2 e^{-ct} e^{-\sqrt{c^2-\omega_0^2}t}$$

$$= e^{-ct}\left\{ A_1 e^{\sqrt{c^2-\omega_0^2}t} + A_2 e^{-\sqrt{c^2-\omega_0^2}t} \right\} \text{——— (6)}$$

$A_1, A_2 \rightarrow$ constant

Damped harmonic oscillation depend on c and ω.

$c > \omega_0 \Rightarrow$ over damped case.

$c = \omega_0 \Rightarrow$ critically damped case.

$c < \omega_0 \Rightarrow$ under damped case.

In overdamped case

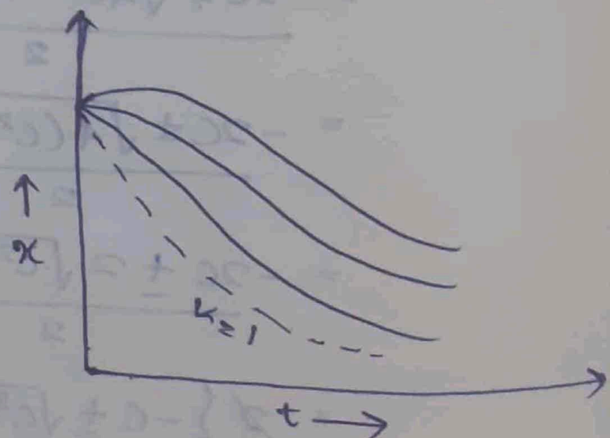
$c > \omega_0$

$\sqrt{c^2-\omega_0^2}$ is +ve

$\therefore \sqrt{c^2-\omega_0^2} = \beta$

$$(6) \Rightarrow x = e^{-ct}\left\{ A_1 e^{\beta t} + A_2 e^{-\beta t} \right\}$$

$$= A_1 e^{(-c+\beta)t} + A_2 e^{(-c-\beta)t}$$



Here the RHS decreases exponentially with time without change in direction. This motion is known as Non - oscillatory. (Dead beat / aperiodic).

eg: Dead beat galvanometer

In critically damped case

$c = \omega_0$

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(6) $\Rightarrow$ $x = A_1 e^{-ct} + A_2 e^{-ct}$,

here $c^2 - \omega_0^2 = 0$

ie, $x = (A_1 + A_2) e^{-ct}$

A_1 and A_2 are constant, so constant + constant = constant.

$\therefore A_1 + A_2 = B$

$$x = B e^{-ct}$$

we use,

$\sqrt{c^2 - \omega_0^2} = h$, but $h \to 0$.

$$x = A_1 e^{(-c + \sqrt{c^2 - \omega_0^2})t} + A_2 e^{(-c - \sqrt{c^2 - \omega_0^2})t}$$

$$= A_1 e^{(-c+h)t} + A_2 e^{(-c-h)t}$$

$e^{-x} = 1 - x$

$e^{x} = 1 + x$

$$= e^{-ct}\left(A_1 e^{ht} + A_2 e^{-ht}\right)$$

$$= e^{-ct}\left\{A_1(1+ht) + A_2(1-ht)\right\}$$

$$= e^{-ct}\left(A_1 + A_1 ht + A_2 - A_2 ht\right)$$

$$= e^{-ct}\left\{(A_1 + A_2) + ht(A_1 - A_2)\right\}$$

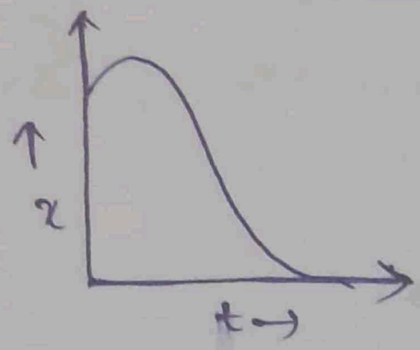
$$= e^{-ct}\left\{D + Et\right\}$$

here we use

$A_1 + A_2 = D$

$(A_1 - A_2)h = E$

Initially displacement increases due to $(D + Et)$. then finally reaches to zero.



in under damped case

$c < \omega_0$,

$$\sqrt{c^2-\omega_0^2} = \sqrt{-(\omega_0^2-c^2)} = i\omega$$

$$e^{i\theta} = \cos\theta + i\sin\theta$$
$$e^{-i\theta} = \cos\theta - i\sin\theta$$

ie, $\omega_0^2 - c^2 = \omega$

$\sqrt{-1} = i$

$$(6) \Longrightarrow x = e^{-ct}\left[A_1 e^{i\omega t} + A_2 e^{-i\omega t}\right]$$

$$= e^{-ct}\left[A_1(\cos\omega t + i\sin\omega t) + A_2(\cos\omega t - i\sin\omega t)\right]$$

$$= e^{-ct}\left(A_1\cos\omega t + A_1 i\sin\omega t + A_2\cos\omega t - A_2 i\sin\omega t\right)$$

$$= e^{-ct}\left((A_1+A_2)\cos\omega t + (A_1-A_2)i\sin\omega t\right)$$

let,

$A_1 + A_2 = a_0\sin\phi$

$i(A_1 - A_2) = a_0\cos\phi$

then,

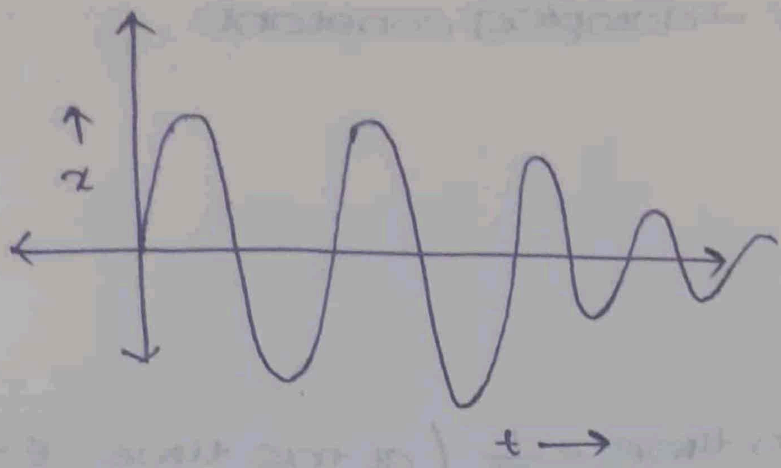
$$x = e^{-ct}(a_0\sin\phi\cos\omega t + \sin\omega t\, a_0\cos\phi)$$

$$= e^{-ct}a_0(\sin\phi\cos\omega t + \sin\omega t\cos\phi)$$

$$= e^{-ct}a_0\sin(\omega t + \phi).$$

similar to simple harmonic oscillator

frequency ↓ time ↑



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(*) The Amplitude of oscillation decreases exponentially with time

(*) Angular frequency of oscillation decreases and period of oscillation increases.

$$\omega = \sqrt{\omega_0^2 - c^2}, \quad \omega < \omega_0$$

$$T = \frac{2\pi}{\omega}$$

$$= \frac{2\pi}{\sqrt{\omega_0^2 - c^2}} > \frac{2\pi}{\omega_0}$$

ENERGY OF DAMPED HARMONIC OSCILLATOR

Energy of damped harmonic oscillator is directly proportional to square of Amplitude.

we use,

$$a_0 e^{-ct} = a$$

$$E \propto a^2$$

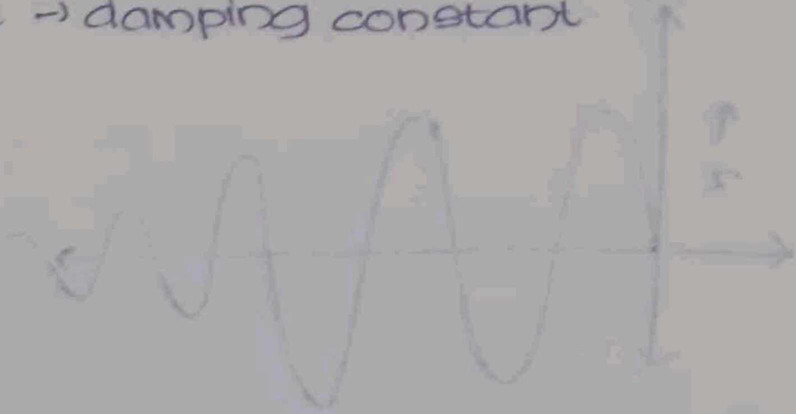
$$E \propto (a_0 e^{-ct})^2$$

$$E \propto a_0^2 e^{-2ct}$$

$$E = E_0 e^{-2ct}.$$

E_0 = Initial energy of damped harmonic oscillator. at $t = 0$

when $t = \frac{1}{2c}$, $c \rightarrow$ damping constant



$$E = E_0 e^{-2c \times \frac{1}{2c}}$$

$$= E_0 e^{-1}$$

$$E = \frac{E_0}{e}$$

$\therefore \tau$, relaxation time $= \frac{1}{2c}$ (at this time $E = 1/e$ of E_0).

QUALITY FACTOR OF DAMPED HARMONIC OSCILLATOR

$$Q = 2\pi \frac{\text{Energy stored}}{\text{Energy lost per period}}$$

$$E_t = E_0 e^{-2ct}$$

$$E_{t+T} = E_0 e^{-2c(t+T)}$$

$$= E_0 e^{-2ct + -2cT}$$

$$= E_0 e^{-2ct} e^{-2cT}$$

$$= E_t e^{-2cT}$$

$$\Delta E_T = E_t - E_{t+T}$$

$$= E_t - E_t e^{-2cT}$$

$$= E_t (1 - e^{-2cT})$$

$$= E_t (1 - (1 - 2cT))$$

$$= E_t (1 - 1 + 2cT)$$

$$= E_t \, 2cT$$

$$\therefore Q = \cancel{2\pi \frac{E_t}{E_t 2CT}}$$

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$$Q = 2\pi \frac{E_t}{E_t\, 2CT}$$

$$= \frac{2\pi}{T} \times \frac{1}{2C}$$

$$\boxed{Q = \omega_0 \tau}$$

$\frac{2\pi}{T} = \omega_0$

$\frac{1}{2C} = \tau$, relaxation time.

$$\omega_0 = \sqrt{\frac{K}{M}}, \quad \tau = \frac{1}{2C}$$

$$= \frac{1}{\cancel{2} \times \frac{r}{\cancel{2}M}} = \frac{M}{r}$$

$$Q = \sqrt{\frac{K}{M}} \times \frac{M}{r}$$

$$= \sqrt{\frac{KM^2}{M}} \times \frac{1}{r}$$

$$= \frac{\sqrt{KM}}{r}$$

PRACTICAL CASE OF DAMPING

(i) Guitar

(ii) Automobile suspension.

FOR LCR CIRCUIT, FREQUENCY OF OSCILLATION :-

$$= \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

$$Q = \frac{L\omega}{R}$$

FORCED OR DRIVEN HARMONIC OSCILLATOR

Restoring Force, R.F $= -cx$.

Damping Force, DF $= -r\cdot\frac{dx}{dt}$.

Driven $= F_0 \sin pt$.

$$m\frac{d^2x}{dt^2} = -cx - r\frac{dx}{dt} + F_0 \sin pt$$

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + cx = F_0 \sin pt$$

divide by m,

$$\frac{d^2x}{dt^2} + \frac{r}{m}\frac{dx}{dt} + \frac{c}{m}x = \frac{F_0}{m}\sin pt.$$

$\frac{F_0}{m} = f_0$

$$\Longrightarrow \frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega_0^2 x = f_0 \sin pt \quad \text{——(1)} \qquad \left[\frac{r}{2m} = k\right]$$

This is a linear differential equation of second order and it's solution contain 2 parts.

(i) complementary Function

it is a solution of $\frac{d^2x}{dt^2} + 2k\frac{dx}{dt} + \omega_0^2 x = 0$

$\Rightarrow$ solution,

$$x = a_0 e^{-kt} \sin(\omega t + \phi)$$

where $\omega = \sqrt{\omega_0^2 - k^2}$

(ii)

$$x = A\sin(Pt-\phi) \quad \text{——— (a)}$$

$$\frac{dx}{dt} = A\cos(Pt-\theta)\frac{d}{dt}(Pt-\theta)$$

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$$= PA\cos(Pt-\theta) \quad \text{——— (c)}$$

$$\frac{d^2x}{dt^2} = PA - \sin(Pt-\theta)\frac{d}{dt}(Pt-\theta)$$

$$= -P^2A\sin(Pt-\theta) \quad \text{——— (b)}$$

(c), (a) and (b) ——— (1)

$$-P^2A\sin(Pt-\theta) + 2K\,PA\cos(Pt-\theta) + \omega_0^2A\sin(Pt-\theta) = f_0\sin pt.$$

$$A(\omega_0^2-p^2)\sin(Pt-\theta) + 2KAP\cos(Pt-\theta) = f_0\sin pt.$$

$$A(\omega_0^2-p^2)\sin(Pt-\theta) + 2KAP\cos(Pt-\theta) = f_0\sin(Pt-\theta+\theta).$$

$$\Rightarrow A(\omega_0^2-p^2)\sin(Pt-\theta) + 2KAP\cos(Pt-\theta) = f_0\sin(Pt-\theta)\cos\theta + f_0\cos(Pt-\theta)\sin\theta.$$

Equating the coefficients of $\sin(Pt-\theta)$ and $\cos(Pt-\theta)$.

$$A(\omega_0^2-p^2) = f_0\cos\theta \quad \text{——— (2)}$$

$$2KAP = f_0\sin\theta \quad \text{——— (3)}$$

Squaring and adding (2) and (3)

$$A^2(\omega_0^2-p^2)^2 + 4K^2A^2p^2 = f_0^2\cos^2\theta + f_0^2\sin^2\theta.$$

$$= f_0^2(\cos^2\theta+\sin^2\theta)$$

$$= f_0^2$$

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$$\Rightarrow f_0^2 = A^2(\omega_0^2-p^2)^2 + 4k^2A^2p^2$$

$$= A^2\{(\omega_0^2-p^2)^2 + 4k^2p^2\}$$

$$A^2 = \frac{f_0^2}{(\omega_0^2-p^2)^2+4k^2p^2}$$

$$\boxed{A = \frac{\sqrt{f_0}}{\sqrt{(\omega_0^2-p^2)^2+4k^2p^2}} \quad \text{——— (Z)}}$$

$(3)/(2)$ - $\tan\theta = \dfrac{2kAp}{A(\omega_0^2-p^2)}$

$$\tan\theta = \frac{2kp}{(\omega_0^2-p^2)}$$

$$\cancel{\frac{f_0}{(\omega_0^2-p^2)+2kp}}$$

substitute $A = \dfrac{f_0}{\sqrt{(\omega_0^2-p^2)^2+4k^2p^2}}$ in (a)

$$x = \frac{f_0}{\sqrt{(\omega_0^2-p^2)^2+4k^2p^2}}\sin(pt-\theta) \quad \text{——— (4)}$$

$\therefore$ the complete solution becomes,

$$x = a_0e^{-kt}\sin(\omega t+\phi) + \frac{f_0}{\sqrt{(\omega_0^2-p^2)^2+4k^2p^2}}\sin(pt-\theta) \quad \text{——— (6)}$$

Initially both the vibrations will be present, but with the passage of time, first term vanishes and motion of the body will be completely represented by the second term.

$$x = \frac{f_0}{\sqrt{(\omega_0^2-p^2)^2+4k^2p^2}}\sin(pt-\theta) \quad \text{——— (6).}$$

AMPLITUDE RESONANCE

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4k^2p^2}}$$

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$\rightarrow (\omega_0^2 - p^2)^2 + 4k^2p^2 = 0$, $A = A_{max}$

$$\frac{d}{dp}\left[(\omega_0^2 - p^2)^2 + 4k^2p^2\right] = 2(\omega_0^2 - p^2)\frac{d}{dp} - p^2 + 4k^2 2p = 0$$

$$= -4p(\omega_0^2 - p^2) + 8k^2p = 0$$

$$4p\left[(-(\omega_0^2 - p^2)) + 2k^2\right] = 0$$

$$-\omega_0^2 + p^2 + 2k^2 = 0$$

$$p^2 = \omega_0^2 - 2k^2$$

$$p = \sqrt{\omega_0^2 - 2k^2}$$

$\Longrightarrow P_R = P = \sqrt{\omega_0^2 - 2k^2}$, where P_R is resonance frequency.

$$A_{max} = \frac{f_0}{\sqrt{(\omega_0^2 - P_R^2)^2 + 4k^2P_R^2}}$$

$p^2 = \omega_0^2 - 2k^2$
$2k^2 = \omega_0^2 - p^2$

$$= \frac{f_0}{\sqrt{(2k^2)^2 + 4k^2P_R^2}}$$

$$= \frac{f_0}{\sqrt{4k^4 + 4k^2P_R^2}}$$

$$= \frac{f_0}{2k\sqrt{k^2 + P_R^2}}$$

At low damping K^2 can be neglected

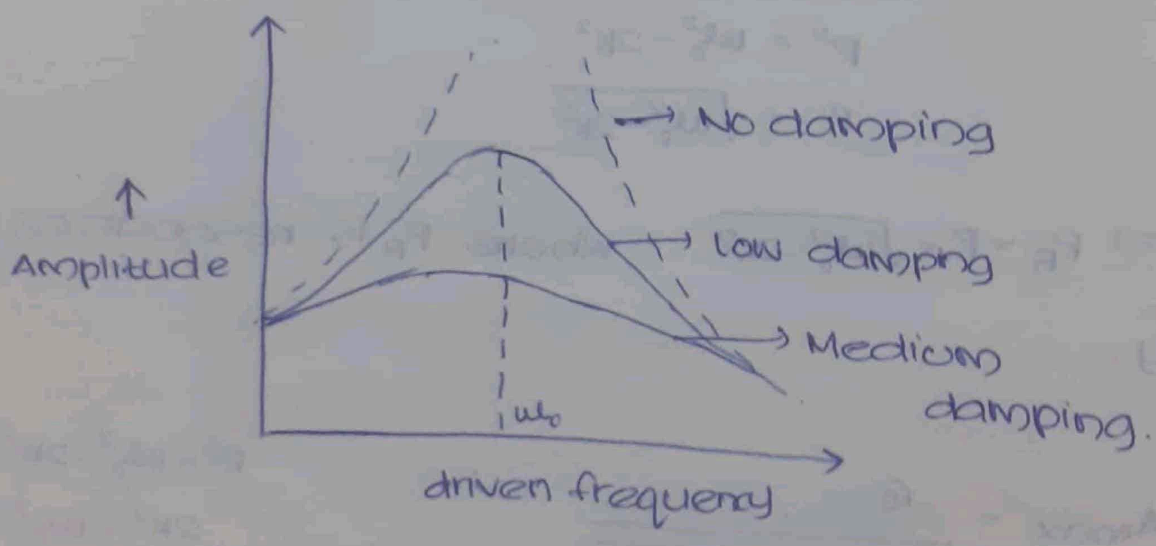
$$P_R = \sqrt{\omega_0^2 - 2K^2}$$

(i) $P_R^2 = \omega_0^2$

$$A_{max} = \frac{f_0}{2KP_R} \qquad \frac{1}{2K} = \tau = \text{relaxation time}$$

$$= \frac{f_0}{2K\omega_0}$$

$$= \frac{f_0 \tau}{\omega_0}$$



NOTE :-

when driven frequency is much less than ω_0

$$P << \omega_0$$

from — (2)

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4K^2p^2}}$$

then, $P << \omega_0$

P become neglected.

$$A = \frac{f_0}{\sqrt{(\omega_0^2)^2}}$$

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$$= \frac{f_0}{\omega_0^2}$$

$$f_0 = \frac{F_0}{M}$$

$$= \frac{F_0}{M\omega_0^2}$$

$$C/M = \omega_0^2$$

$$C = M\omega_0^2$$

$$A = \frac{F_0}{C}$$

this shows that Amplitude is a constant.

(ii) $P >> \omega_0$

$$A = \frac{f_0}{p^2}$$

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - p^2)^2 + 4k^2p^2}}$$

here k is very small hence it become zero

$$= \frac{F_0}{Mp^2}$$

QUALITY FACTOR

No. of rotation an oscillator can done before it coming to rest.

$$Q = \frac{A_{max}}{A_{at\ p=0}}$$

Ratio of Amplitude at resonance to the Amplitude of zero driven frequency.

$$Q = \frac{f_0/2k\mu\omega_0}{f_0/\mu\omega_0^2}$$

$$= \frac{1}{2k}\omega$$

$$\underline{\underline{Q = \tau\omega}}$$

$$Q = \frac{\sqrt{C/M}}{r/M} \Longrightarrow \underline{\underline{\frac{\sqrt{CM}}{r}}}$$

SHARPNESS OF RESONANCE

The term sharpness of resonance refers to the rate of fall of amplitude with change in driven frequency on either side of the resonance frequency. We see that when damping is small, the amplitude falls of rapidly on either side of the resonance frequency. Then we say the resonance is sharp.

When damping is large, the amplitude falls of very slowly on either side of the resonance frequency, then we say the resonance is flat.

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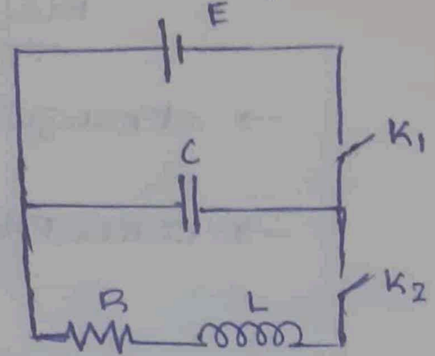
LCR CIRCUIT AS AN ELECTRICAL ANALOG OF MECHANICAL OSCILLATOR

$\rightarrow E = V_0 \sin \omega t$



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potential difference across inductor :-

$$V_L = L\frac{di}{dt} \qquad i = \frac{dq}{dt},\ q = \text{charge}$$

$$= L\frac{d^2q}{dt^2}$$

potential difference across resistor :-

$$V_R = iR = R\frac{dq}{dt}$$

potential difference across capacitor :-

$$V_C = \frac{q}{C}$$

$\Longrightarrow \quad V_L + V_R + V_C = E$

ie, $V_0 \sin \omega t = V_L + V_R + V_C$

$$V_0 \sin \omega t = L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + q/C$$

MECHANICAL	ELECTRICAL.
→ Displacement x	→ charge q.
→ velocity $v = \frac{dx}{dt}$	→ current $i = \frac{dq}{dt}$
→ Mass M	→ inductance L
→ Damping coefficient r	→ Resistance R
→ Force constant k	→ Resiprocal of capacitance $\frac{1}{C}$
→ potential energy $\frac{1}{2}kx^2$	→ potential energy = energy stored in capacitor $\frac{1}{2}CV^2$.
→ kinetic energy $\frac{1}{2}MV^2$	→ $\frac{1}{2}LI^2$
→ Resonance frequency $\nu_0 = \frac{1}{2\pi}\sqrt{\frac{C}{M}}$	→ $\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

WAVES

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ONE DIMENSIONAL WAVEEQUATION;

wave function in +ve direction, $\psi_{(x,t)} = f(x-vt)$ — (1)

-ve direction, $\psi_{(x,t)} = f(x+vt)$ — (2)

$$\Longrightarrow \psi_{(x,t)} = f(x \pm vt)$$

Differential equation

differential eqn (1) with respect to x, twice.

$$\psi_{(x,t)} = f(x-vt)$$

$$\frac{\partial \psi}{\partial x} = f'(x-vt)$$

$$\frac{\partial^2 \psi}{\partial x^2} = f''(x-vt) \quad \text{— (3).}$$

differential eqn (1) with respect to t, twice

$$\frac{\partial \psi}{\partial t} = f'(x-vt) \times -v$$

$$\frac{\partial^2 \psi}{\partial t^2} = v^2 f''(x-vt). \quad \text{— (4)}$$

$$(3) \& (4) \Longrightarrow \frac{\partial^2 \psi}{\partial t^2} = v^2 \frac{\partial^2 \psi}{\partial x^2}$$

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}$$

This differential equation of a one dimensional wave.

Generally, solution of a wave is

$$\psi_{(x,t)} = A \sin k(x - vt)$$

$k \rightarrow$ propogation constant

DEFINITION OF WAVELENGTH AND PERIOD:-

$$\psi(x,t) = A \sin k(x - vt).$$

replace x with $x + \frac{2\pi}{k}$

$$\psi(x,t) = A \sin k\left(x + \frac{2\pi}{k} - vt\right)$$

$$= A \sin\left[kx + 2\pi - vtk\right]$$

$$= A \sin\left[k(x - vt) + 2\pi\right]$$

$$= A \sin\left[k(x - vt)\right]$$

$\sin(\theta + 2\pi) = \sin\theta.$

This means $\psi(x,t)$ represents a periodic wave with it's space periodicity $\frac{2\pi}{k}$, ie the wave has the same value of displacement at x and $x + \frac{2\pi}{k}$.

Again replace t with $t + \frac{2\pi}{kv}$

$\psi(x,t) = A \sin k(x - vt)$

$$\psi\left(x, t+\frac{2\pi}{kv}\right) = A \sin k\left(x - v\left(t+\frac{2\pi}{kv}\right)\right)$$

$$= A \sin\left[kx - kvt + 2\pi\right]$$

$$= A \sin\left(k\left[(x - vt)\right] + 2\pi\right)$$

$$= A \sin\left[k(x - vt)\right]$$

$$= \psi(x,t).$$

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This means $\psi(x,t)$ represents a periodic wave with time periodicity $\frac{2\pi}{kv}$,

ie this time interval after which the wave has the same value of displacement is known as period of wave.

$T = \frac{2\pi}{kv}$ also $T = \frac{2\pi}{\omega}$

but $k = \frac{2\pi}{\lambda}$

$\omega = kv$

$\frac{2\pi}{kv} = T$

$1/T = \nu$

$\frac{2\pi}{T} = kv$

$\nu = \frac{\omega}{2\pi}$

$\omega = kv$

$2\pi\nu = \omega$

$\therefore 2\pi\nu = kv$

$2\pi\nu = \frac{2\pi}{\lambda} v$

$\nu = \frac{v}{\lambda}$

$\boxed{v = \nu\lambda}$

DIFFERENTIAL EQUATION OF THREE DIMENSIONAL WAVE AND IT'S SOLUTION

$\psi = f(x, y, z, t)$

$$\frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} + \frac{\partial^2\psi}{\partial z^2} = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2} \qquad \left[\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]$$

$$\nabla^2\psi = \frac{1}{v^2}\frac{\partial^2\psi}{\partial t^2}$$

∴ solution is

$$\psi = a e^{\pm i[(k.r) \pm \omega t]}$$

where k is propagation vector

r is direction vector

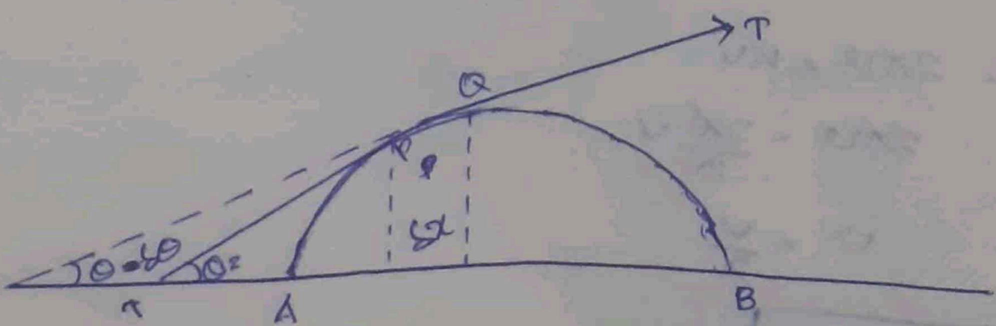
$r = x\hat{i} + y\hat{j} + z\hat{k}$

for waves travelling in the +ve x direction the above solution can be written as in terms of sin or cosine functions.

$\psi = a \sin(k.r - \omega t)$.

$\psi = a \cos(k.r - \omega t)$

TRANSVERSE VIBRATIONS OF A STRETCHED STRING



Consider a small element PQ of length δx, the tangent at P and Q make $\angle\theta$ and $\angle(\theta-\delta\theta)$.

The downward component of tension at P = $T\sin\theta$;

$$\sin\theta = \frac{P}{T}$$

$$\therefore P = T\sin\theta$$

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[When θ is small, $\sin\theta = \tan\theta$]

$\tan\theta$ = slope

$= \frac{dy}{dx}$

$$\Rightarrow \quad P = T\tan\theta \text{ ——— (1)}$$

$$= T\frac{dy}{dx} \text{ ——— (2)}$$

The rate of change of slope with respect to the length of the element.

$$P = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$$

$\therefore$ change in slope for a distance δx is,

$$= \delta x\frac{d^2y}{dx^2}$$

→ slope at the point Q,

$$\tan(\theta-\delta\theta) = \frac{dy}{dx} - \delta x\frac{d^2y}{dx^2}$$

the upward component of tension at Q is :-

$$= T\sin(\theta - \delta\theta)$$

$$= T\tan(\theta - \delta\theta).$$

$$= T\left[\frac{dy}{dx} - \delta x \frac{d^2y}{dx^2}\right]$$

∴ the resultant downward tension F =,

$$F = T\frac{dy}{dx} - T\left[\frac{dy}{dx} - \frac{d^2y}{dx^2}\delta x\right]$$

$$= T\cancel{\frac{dy}{dx}} - T\cancel{\frac{dy}{dx}} + T\frac{d^2y}{dx^2}\delta x$$

$$= T\frac{d^2y}{dx^2}\delta x \text{ ——— (A)}$$

let m be the mass of the unit length of the string

Mass of the element = $m\delta x$

the force acting on the element = mass × acceleration

$$F = m\delta x \times \frac{d^2y}{dt^2} \text{ ——— (5)}$$

(A) & (5) $\Longrightarrow$ $T\frac{d^2y}{dx^2}\cancel{\delta x} = m\cancel{\delta x}\frac{d^2y}{dt^2}$

$$m\frac{d^2y}{dt^2} = T\frac{d^2y}{dx^2}$$

$$\frac{d^2y}{dt^2} = \frac{T}{m}\frac{d^2y}{dx^2} \text{ ——— (6)}$$

This is similar to one dimensional wave equation

$$\frac{d^2\psi}{dt^2} = v^2 \frac{d^2\psi}{dx^2} \quad \text{——— (7)}$$

(6) and (7) $\Longrightarrow$ $v^2 = \frac{T}{m}$

$$v = \sqrt{\frac{T}{M}}$$

v - velocity of propagation of wave.

If ν is the fundamental frequency of vibration of the string.

$$v = \nu\lambda.$$

$$\sqrt{T/M} = \nu\lambda$$

$$\boxed{\nu = \frac{1}{\lambda}\sqrt{T/m}}$$

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In the case of string of length l vibrating in one segment.

$$l = \frac{\lambda}{2}$$

$$\lambda = 2l$$

$$\Longrightarrow \nu = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

LAWS OF TRANSVERSE VIBRATIONS

1. LAW OF LENGTH :-

The fundamental frequency of vibration is inversely proportional to the resonating length of the string.

2. LAW OF TENSION

The fundamental frequency of vibration is directly proportional to the square root of the tension of string.

3. LAW OF MASS.

The fundamental frequency of vibration is inversely proportional to the square root of the mass per length of string.

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